DIFFUSION AND HEAT TRANSFER IN A MULTICOMPONENT COMPLETELY

IONIZED PLASMA

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In a laboratory plasma, particularly in a high-temperature plasma, impurities from the walls of the discharge chamber, diaphragms, etc., play an important role. Even a small fraction of impurity ions can considerably affect the radiation, the electrical conductance, and other plasma parameters. It is therefore important to investigate the diffusion of impurities, and their time and spatial distributions [1, 2].

At high temperatures the atoms of impurity are multiply ionized, so that the problem reduces to investigating transfer processes in a completely ionized multicomponent plasma, formed from electrons and several sorts of ions of arbitrary charge and mass.

Classical diffusion in a plasma consisting of electrons and ions of two sorts has been considered in [3]. An analysis carried out on the basis of the quasihydrodynamic approximation, i.e., neglecting the contribution of thermal forces in the equation of motion for the individual plasma components, has shown that when the ions diffuse transverse to a strong magnetic field ions with high charge should be concentrated in the high-density region of the plasma. A similar conclusion was reached independently in [4]. However, in these papers the effect on diffusion of the transverse temperature gradient was neglected, although the latter is always present under practical conditions. For the special case of a plasma with a small impurity density and when $m_{I} \gg m_{i}$ (m_I and m_{i} are the masses of the impurity ions and of the plasma ions, respectively) diffusion of impurities taking the temperature gradient into account was considered in [5] as it applies to the problem of the "wall" or "nonmagnetic" trapping of a plasma. In [6] the longitudinal forces of friction and the heat flow in a plasma with two sorts of ions of arbitrary mass were calculated in connection with an analysis of the transfer of impurities in colloidal systems. Consideration of the thermal forces in this case enabled the effect of temperature screening of the impurities to be analyzed.

In this paper we obtain a general system of equations for determining the diffusion velocities and heat flows in a multicomponent plasma with ions of arbitrary mass in arbitrary charge states. Grad's method [7] is used to solve the linearized Boltzmann equation. The transfer equations obtained in [8-10] in the Grad 13-moments approximation, does not give the required accuracy, so here, as in [6], we use the higher approximation corresponding in accuracy to the calculations carried out in [3] for a simple plasma.

We calculate the particle and heat fluxes transverse to the magnetic field in a multicomponent magnetized plasma, which enables us, in particular, to analyze completely the effect of temperature screening, and also the transverse transfer of heat in a plasma with several sorts of ions. When determining the longitudinal properties, the transfer is based on the possibility of summing over the charge states of ions of one sort, which considerably simplifies the calculation of the kinetic coefficients for this case. The expressions obtained, together with the coefficients calculated in Appendix 2 for a plasma with two sorts of ions, can be used to analyze the radial transfer of impurities (in particular, carbon, oxygen, tungsten, and iron impurities) in colloidal systems in the Pfirsh-Schlüter regime (see also [6, 11]).

1. General System of Equations. To determine the diffusion velocities and heat flows in a multicomponent plasma we used Grad's method [7]. The distribution function is represented in this case in the form of an expansion in irreducible Hermite polynomials [12]

$$f_{\alpha z} = f_{\alpha z}^{(0)} = \sum_{n,m=0}^{\infty} \frac{(2m+1)! (m+n)!}{n! (m!)^2 (2n+2m+1)!} \mathbf{a}_{\alpha z}^{mn} \mathbf{H}^{mn} \left(\mathbf{c} \sqrt{\frac{m_{\alpha}}{T_{\alpha z}}} \right), \tag{1}$$

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where $f_{\alpha z}^{(0)} = n_{\alpha z} \left(\frac{m_{\alpha}}{2\pi T_{\alpha z}}\right)^{3/2} \exp\left(-\frac{m_{\alpha}c^2}{2T_{\alpha z}}\right)$ is the local Maxwell distribution function, $\mathbf{c} = \mathbf{v} - \mathbf{u}; \mathbf{U}$ is

the mean-mass velocity of the mixture, m_α is the mass of the particles, and $n_{\alpha z}$ and $T_{\alpha z}$ are the density and temperature of the particles of sort α and charge z. In this expansion m represents the rank of the tensor, while n represents the degree of the polynomial.

In the general case substitution of the expansion (1) into the kinetic equation leads, after integration over the velocity with weight H^{mn} , to a system of nonlinear differential equations for the coefficients $a_{\overline{\alpha}\overline{\alpha}}$ [7, 9]. The latter can be simplified considerably if we assume that the macroscopic parameters of the plasma vary only slightly at distances of the order of the effective mean free path and in a time of the order of the time between collisions between the plasma particles. When these conditions are satisfied we can neglect the derivatives of the coefficients $a_{\alpha z}^{mn}$ and nonlinear terms on the left and right sides of the equations. We finally arrive at a linear system of algebraic equations for $a_{\alpha z}^{mn}$ [8-10].

When considering diffusion and heat transfer we keep only terms with m = 1 in expansion (1). For a fairly accurate calculation of the transfer coefficients in a completely ionized plasma it is necessary to use not less than three terms [3]. In this case it is convenient to transfer from the coefficients $\mathbf{a}_{\alpha z}^{10}$, $\mathbf{a}_{\alpha z}^{11}$ and $\mathbf{a}_{\alpha z}^{12}$ to the moments $\rho_{\alpha z} \mathbf{w}_{\alpha z} = \rho_{\alpha z} (\mathbf{u}_{\alpha z} - \mathbf{u})$, $\mathbf{h}_{\alpha z} = \mathbf{q}_{\alpha z} - 5p_{\alpha z} \mathbf{w}_{\alpha z}/2$ and $\mathbf{r}_{\alpha z}$ as given by the relations

$$\mathbf{a}_{lpha 2}^{10}=
ho_{lpha 2}\mathbf{w}_{lpha 2}\mathbf{\gamma}_{lpha z}^{-1/2}/p_{lpha}, \ \ \mathbf{a}_{lpha z}^{11}=2\mathbf{h}_{lpha 2}\mathbf{\gamma}_{lpha z}^{1/2}/p_{lpha}, \ \ \mathbf{a}_{lpha z}^{12}=4\mathbf{r}_{lpha z}\mathbf{\gamma}_{lpha z}^{3/2}/p_{lpha z},$$

where $\mathbf{u}_{\alpha z}$ and $\mathbf{q}_{\alpha z}$ are the velocity and heat flux of particles of sort α and charge z, $\rho_{\alpha z} = m_{\alpha} n_{\alpha z}$; $p_{\alpha z} = n_{\alpha z} T_{\alpha z}$; $\gamma_{\alpha z} = m_{\alpha}/T_{\alpha z}$. The equations for $w_{\alpha z}$, $h_{\alpha z}$, and $r_{\alpha z}$ can then be written in the form

$$-\rho_{\alpha z}\omega_{\alpha z}\left[\mathbf{w}_{\alpha z}\times\mathbf{k}\right]+\nabla p_{\alpha z}-\rho_{\alpha z}\left(\frac{\mathbf{F}_{\alpha z}}{m_{\alpha}}-\frac{d\mathbf{u}}{dt}\right)=\sum_{\boldsymbol{\beta},\boldsymbol{\zeta}}\left\{G^{(1)}_{\alpha z}\beta_{\boldsymbol{\zeta}}\left(\mathbf{w}_{\alpha z}-\mathbf{w}_{\boldsymbol{\beta}\boldsymbol{\zeta}}\right)+\frac{\mu_{\alpha \beta}}{T}G^{(2)}_{\alpha z}\beta_{\boldsymbol{\zeta}}\left(\frac{\mathbf{h}_{\alpha z}}{\rho_{\alpha z}}-\frac{\mathbf{h}_{\boldsymbol{\beta}\boldsymbol{\zeta}}}{\rho_{\boldsymbol{\beta}\boldsymbol{\zeta}}}\right)+\frac{\mu_{\alpha \beta}^{2}}{T^{2}}G^{(3)}_{\alpha z\beta \boldsymbol{\zeta}}\left(\frac{\mathbf{r}_{\alpha z}}{\rho_{\alpha z}}-\frac{\mathbf{r}_{\boldsymbol{\beta}\boldsymbol{\zeta}}}{\rho_{\boldsymbol{\beta}\boldsymbol{\zeta}}}\right)\right];$$
(2a)

$$-\omega_{\alpha z} \left[\mathbf{h}_{\alpha z} \times \mathbf{k}\right] + \frac{5}{2} \frac{p_{\alpha z}}{m_{\alpha}} \nabla T_{\alpha z} = \frac{T}{m_{\alpha}} \sum_{\beta, \zeta} \left\{ \frac{5}{2} \frac{\mu_{\alpha \beta}}{m_{\alpha}} G_{\alpha z \beta \zeta}^{(2)} \left(\mathbf{w}_{\alpha z} - \mathbf{w}_{\beta \zeta}\right) + G_{\alpha z \beta \zeta}^{(4)} \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}} - G_{\alpha z \beta \zeta}^{(5)} \frac{\mathbf{h}_{\beta \zeta}}{p_{\beta \zeta}} + \frac{\mu_{\alpha \beta}}{T} \left(G_{\alpha z \beta \zeta}^{(6)} \frac{\mathbf{r}_{\alpha z}}{p_{\alpha z}} - G_{\alpha z \beta \zeta}^{(7)} \frac{\mathbf{r}_{\beta \zeta}}{p_{\beta \zeta}} \right) \right\};$$
(2b)

$$-\omega_{\alpha z} \left[\mathbf{r}_{\alpha z} \times \mathbf{k} \right] = \frac{T^2}{m_{\alpha}^2} \sum_{\beta, \zeta} \left\{ \frac{35}{2} \left(\frac{\mu_{\alpha \beta}}{m_{\alpha}} \right)^2 G_{\alpha z \beta \zeta}^{(3)} \left(\mathbf{w}_{\alpha z} - \mathbf{w}_{\beta \zeta} \right) + 7 \frac{\mu_{\alpha \beta}}{m_{\alpha}} \left(G_{\alpha z \beta \zeta}^{(6)} \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}} - G_{\alpha z \beta \zeta}^{(7)} \frac{\mathbf{h}_{\beta \zeta}}{p_{\beta \zeta}} \right) + G_{\alpha z \beta \zeta}^{(8)} \frac{m_{\alpha} \mathbf{r}_{\alpha z}}{T p_{\alpha z}} - G_{\alpha z \beta \zeta}^{(9)} \frac{m_{\beta} \mathbf{r}_{\beta \zeta}}{T p_{\beta \zeta}} \right],$$
(2c)

where $\mathbf{F}_{\alpha z} = \mathbf{X}_{\alpha} + ez \left(\mathbf{E} + [\mathbf{u} \times \mathbf{H}]/c\right)$, \mathbf{X}_{α} are forces of nonelectromagnetic origin, $\omega_{\alpha z} = ezH/m_{\alpha}c$; $\mu_{\alpha\beta} = m_{\alpha}m_{\beta}/(m_{\alpha} + m_{\beta})$; $\mathbf{k} = \mathbf{H}/\mathbf{H}$. The index relates to the charged state of the ions of sort β . The quantities $\mathbf{G}_{\alpha z \beta \xi}^{(\mathbf{n})}$, taking into account the relationship between the irreducible

Hermite polynomials H^{1k} and the Sonin polynomials $S_{3/2}^{(k)}[12]$

$$\mathbf{H}^{\mathbf{1}\mathbf{k}}(u) = (-2)^{k} k! S_{\mathbf{3}/2}^{(k)} \left(\frac{u^{2}}{2}\right) \mathbf{u}$$

can be expressed in terms of the known integral brackets of the Sonin polynomials [13, 14]. Calculation of the latter for a completely ionized plasma leads to the following results:

$$G_{\alpha z\beta \zeta}^{(1)} = -W_{\alpha z\beta \zeta}, \quad G_{\alpha z\beta \zeta}^{(2)} = \frac{3}{5} W_{\alpha z\beta \zeta}, \quad G_{\alpha z\beta \zeta}^{(3)} = -\frac{3}{14} W_{\alpha z\beta \zeta}, \tag{3}$$

$$G_{\alpha z\beta \zeta}^{(4)} = -\left(\frac{13}{10}\frac{m_{\beta}}{m_{\alpha}} + \frac{8}{5} + 3\frac{m_{\alpha}}{m_{\beta}}\right) \varkappa_{\alpha \beta} W_{\alpha z\beta \zeta}, \quad G_{\alpha z\beta \zeta}^{(5)} = -\frac{27}{10} \varkappa_{\alpha \beta} W_{\alpha z\beta \zeta},$$

$$G_{\alpha z\beta \zeta}^{(6)} = \frac{3}{5} \left(\frac{23}{28} \frac{m_{\beta}}{m_{\alpha}} + \frac{8}{7} + 3 \frac{m_{\alpha}}{m_{\beta}} \right) \varkappa_{\alpha\beta} W_{\alpha z\beta \zeta}, \quad G_{\alpha z\beta \zeta}^{(7)} = \frac{45}{28} \varkappa_{\alpha\beta} W_{\alpha z\beta \zeta},$$

$$G_{\alpha z\beta \zeta}^{(8)} = - \left(\frac{433}{280} \frac{m_{\beta}^2}{m_{\alpha}^2} + \frac{139}{35} \frac{m_{\beta}}{m_{\alpha}} + \frac{459}{35} + \frac{32}{5} \frac{m_{\alpha}}{m_{\beta}} + 5 \frac{m_{\alpha}^2}{m_{\beta}^2} \right) \varkappa_{\alpha\beta}^2 W_{\alpha z\beta \zeta},$$

$$G_{\alpha z\beta \zeta}^{(9)} = - \frac{75}{8} \varkappa_{\alpha\beta}^2 W_{\alpha z\beta \zeta},$$
(3)

where

$$\varkappa_{\alpha\beta} = m_{\alpha}m_{\beta}/(m_{\alpha} + m_{\beta})^{2}; \ W_{\alpha z\beta \zeta} = W_{\beta\zeta\alpha z} = n_{\alpha z}m_{\alpha}/\tau_{\alpha z\beta \zeta};$$
$$\tau_{\alpha z\beta \zeta}^{-1} = \frac{4\sqrt{2\pi}}{3} \frac{n_{\beta \zeta} z^{2} \zeta^{2} e^{4} \mu_{\alpha\beta}^{1/2} \lambda}{m_{\alpha} T^{3/2}}.$$

Here λ is the Coulomb logarithm [3] which, due to the weak dependence on the parameters, is assumed to be approximately the same for all sorts of particles.

Note that the right sides of the system of equations (2) was calculated assuming the temperatures of all the sorts of particles to be equal. Otherwise the equations become considerably more complicated (cf., e.g., [9], where the right sides of the equations for the velocities and thermal flux are calculated for arbitrary temperatures of the components). In fact, to preserve the form and values of the coefficients of system (2), strict equality of the temperatures is required, as well as satisfaction of the condition $|T_{\alpha z} - T_{\beta \xi}| \ll T_{\alpha z}$. The approximate equality of the temperatures of the plasma components does not, however, impose any limitations on the temperature gradients. Hence, both in the initial system of equations (2) and in all the results obtained, unless otherwise stated, the temperatures of the components are assumed to be approximately equal, while the temperature gradients are arbitrary.

The system of linear algebraic equations (2) with coefficients (3) enables one to determine the velocity of diffusion and the heat fluxes in a multicomponent completely ionized plasma for arbitrary values of $\omega\tau$. In the special case of a two-component electron—ion plasma the solution of this system in the limit when $m_e/m_i \ll 1$ leads to the results obtained in [3] using the Chapman—Enskog method. For a multicomponent plasma consisting of electrons and ions of different sorts in different charged states, the general solution of system (2) for arbitrary $\omega\tau$ looks fairly complicated, which in practice eliminates the possibility of analyzing it. Hence, below we will consider solutions of Eqs. (2) in the limits when $\omega\tau \gg 1$ and $\omega = 0$, which enables fairly simple expressions to be obtained for the diffusion velocities and heat flows in important practical cases: transverse heat flows and particles of arbitrary charge and mass in a magnetized plasma, longitudinal flows for electrons or light ions, and longitudinal flows for several sorts of heavy ions in arbitrary charged states.

2. Diffusion and Heat Transfer Transverse to a Magnetic Field in a Magnetized Plasma. In a magnetized plasma ($\omega \tau \gg 1$, τ^{-1} is the collision frequency) the solution of Eqs. (2) for components perpendicular to the magnetic field can be obtained using an expansion with respect to the small parameter ($\omega \tau$)⁻¹. In the zeroth approximation, completely neglecting collisions, we have

$$\mathbf{w}_{\alpha z \perp}^{(0)} = \frac{1}{\rho_{\alpha z} \omega_{\alpha z}} \left[\mathbf{k} \times \nabla p_{\alpha z} \right] + \frac{c}{H^2} \left[\mathbf{E} \times \mathbf{H} \right] + \frac{1}{\omega_{\alpha z}} \left[\mathbf{k} \times \frac{d\mathbf{u}}{dt} \right], \tag{4}$$
$$\mathbf{h}_{\alpha z \perp}^{(0)} = \frac{5}{2} \frac{p_{\alpha z} c}{e H z} \left[\mathbf{k} \times \nabla T_{\alpha z} \right], \quad \mathbf{r}_{\alpha z \perp}^{(0)} = 0. \tag{4}$$

The diffusion flows of particles and heat appear in the first approximation with respect to $(\omega\tau)^{-1}$ when Eqs. (4) are substituted into the right side of Eqs. (2). In this case it should be noted that consideration of the moment $\mathbf{r}_{\alpha Z}^{(o)}$ in the expansion, since in a magnetized plasma $\mathbf{r}_{\alpha z\perp}^{(0)} = 0$, does not lead to any refinements compared with the well-known 13moment approximation, corresponding to consideration of only the first two moments in expansion (1). Hence, the results given below can be obtained directly from the general expressions given in [9], in which the derivation of the transfer equations for a nonisothermal multisort plasma was considered in the 13-moment approximation. Note that in [9] the case of a strongly magnetized plasma was not analyzed separately. As regards [15, 16], based on other methods of solution, expressions are derived in them only for the diffusion velocities transverse to a strong magnetic field. It is therefore advisable to present the final expressions for the diffusion velocities and the heat flows which follow from system (2). Neglecting, for simplicity, nonelectromagnetic forces, we obtain

$$\mathbf{w}_{\alpha z \perp} = -\frac{z}{m_{\alpha} \omega_{\alpha}^{2}} \sum_{\beta} \frac{1}{\tau_{\alpha \beta}} \left[\frac{\mathbf{\nabla}_{\perp} p_{\alpha z}}{zn_{\alpha z}} - \frac{T \mathbf{\nabla}_{\perp} (n_{\beta} \tilde{z}_{\beta}) + n_{\beta} \tilde{z}_{\beta} \mathbf{\nabla}_{\perp} \tilde{T}_{\beta}^{(1)}}{n_{\beta} \tilde{z}_{\beta}^{2}} + \left(\frac{m_{\alpha}}{z} - \frac{m_{\beta} \tilde{z}_{\beta}}{z_{\beta}^{2}} \right) \frac{d\mathbf{u}_{\perp}}{dt} - \frac{3}{2} \mu_{\alpha \beta} \left(\frac{\mathbf{\nabla}_{\perp} T_{\alpha z}}{zm_{\alpha}} - \frac{\tilde{z}_{\beta} \mathbf{\nabla}_{\perp} \tilde{T}_{\beta}^{(1)}}{z_{\beta}^{2} m_{\beta}} \right) \right]; \quad (5)$$
$$\mathbf{h}_{\alpha z \perp} = \frac{3}{2} \frac{p_{\alpha z} z}{m_{\alpha}^{2} \omega_{\alpha}^{2}} \sum_{\beta} \frac{\mu_{\alpha \beta}}{\tau_{\alpha \beta}} \left[\frac{\mathbf{\nabla}_{\perp} p_{\alpha z}}{n_{\alpha z} z} - \frac{T \mathbf{\nabla}_{\perp} (n_{\beta} \tilde{z}_{\beta}) + n_{\beta} \tilde{z}_{\beta} \mathbf{\nabla}_{\perp} \tilde{T}_{\beta}^{(1)}}{n_{\beta} \tilde{z}_{\beta}^{2}} + \frac{1}{n_{\beta} \tilde{z}_{\beta}^{2}} \right]$$

$$+\left(\frac{m_{\alpha}}{z}-\frac{m_{\beta}\overline{z}_{\beta}}{\overline{z}_{\beta}^{2}}\right)\frac{d\mathbf{u}_{\perp}}{dt}\right]-\frac{2p_{\alpha z}}{m_{\alpha}\omega_{\alpha}^{2}}\sum_{\beta}\frac{\mu_{\alpha\beta}}{(m_{\alpha}+m_{\beta})\tau_{\alpha\beta}}\left[\left(\frac{43}{8}\frac{m_{\beta}}{m_{\alpha}}+2+\frac{45}{4}\frac{m_{\alpha}}{m_{\beta}}\right)\times\nabla_{\perp}T_{\alpha z}-\frac{27}{8}\frac{\overline{z}_{\beta}}{\overline{z}_{\beta}^{2}}\nabla_{\perp}\overline{T}_{\beta}^{(1)}\right].$$

For convenience we have carried out the summation over the charged states of the ions in Eqs. (5) and (6), so that the sums in these expressions can only be extended to particles with different masses. In the summation we introduced the following notation:

$$\overline{z_{\alpha}^{k}} = \sum_{z} n_{\alpha z} z^{k} / n_{\alpha}, \quad n_{\alpha} = \sum_{z} n_{\alpha z}, \quad p_{\alpha} = \sum_{z} p_{\alpha z},$$

$$T_{\alpha} = p_{\alpha} / n_{\alpha}, \quad \nabla \overline{T}_{\alpha}^{(k)} = \sum_{z} n_{\alpha z} z^{k} \nabla T_{\alpha z} / n_{\alpha} \overline{z_{\alpha}^{k}} \left(\nabla \overline{T}_{\alpha}^{(0)} \equiv \nabla T_{\alpha} \right),$$

$$\tau_{\alpha\beta}^{-1} = \sum_{z,\zeta} n_{\alpha z} / n_{\alpha} \tau_{\alpha z\beta \zeta}, \quad \omega_{\alpha}^{2} = e^{2} H^{2} \overline{z_{\alpha}^{2}} / m_{\alpha}^{2} c^{2}.$$

We will also present an expression for the transverse component of the force of friction of particles of the α -sort

$$\mathbf{R}_{\alpha z \perp} = -\frac{m_{\alpha} n_{\alpha z} z^{2}}{z_{\alpha}^{2}} \sum_{\beta} \frac{1}{\tau_{\alpha \beta}} \left\{ \mathbf{w}_{\alpha z \perp} - \overline{\mathbf{w}}_{\beta \perp} - \frac{3}{2} \frac{c \mu_{\alpha \beta}}{e H} \left(\frac{[\mathbf{k} \times \nabla T_{\alpha z}]}{z m_{\alpha}} - \frac{\overline{z}_{\beta} [\mathbf{k} \times \nabla \overline{T}_{\beta}^{(1)}]}{m_{\beta} \overline{z}_{\beta}^{2}} \right) \right\}, \tag{7}$$

where $\overline{\mathbf{w}}_{\alpha} = \sum_{z} n_{\alpha z} z^2 \mathbf{w}_{\alpha z} / n_{\alpha} \overline{z_{\alpha}^2}$. The summation in (7), as in Eqs. (5) and (6), can only be extended to different sorts of particles.

For illustration we will consider some consequences of Eqs. (5) and (6). We will determine the equilibrium (equilibrium established after a time that is less than the characteristic electron diffusion time [3]) concentration of impurity ions in a plasma consisting of electrons, basic singly charged ions i and ions of sort I with arbitrary charges z. The equations for the equilibrium value n_I in the cylindrically symmetrical case can be obtained from Eq. (5), by equating to zero the radial components of the flux ($\Gamma_I = \Sigma n_{IZ} w_{IZ}$) and the current ($J_I = e \Sigma n_{IZ} w_{IZ} z$) of the impurity ions. Neglecting collisions with electrons and the inertial terms, and also assuming $\bar{z}_I^2 \approx \bar{z}_I^2$, we have

$$\frac{\partial \overline{z}_I}{\partial r} = 0, \quad \frac{1}{n_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_r \overline{z}_I^2} \frac{\partial \left(p_I \overline{z}_I \right)}{\partial r} - \frac{3}{2} \mu_{iI} \left(\frac{1}{m_i} - \frac{1}{m_I \overline{z}_I} \right) \frac{\partial T}{\partial r} = 0.$$
(8)

An important consequence of these equations is the fact that equilibrium is reached only when $\bar{z}_{\rm I}$ = const, i.e., in those regions where the impurity ions are ionized up to the maximum charged state possible for the specified electron temperature. Otherwise, due to collisions between impurity ions of different charge there is a flux given by $\Gamma_I = \frac{\rho_{HI}^2 n_I}{2\tau_{II}} \frac{1}{\bar{z}_{\tau}} \frac{\partial \bar{z}_I}{\partial r}$, where $\rho_{\rm HI}^2 =$

 $2T/m_{I}\omega_{I}^{2}$, directed in the direction in which \bar{z}_{I} increases.* In regions where \bar{z}_{I} =

The comparatively simple form of the expression for Γ_{I} and the equilibrium conditions (8) is due to the assumption made above that $\overline{z_{I}^{2}}$ and $\overline{z_{I}^{2}}$ are approximately equal. In general these expressions turn out to be more complex, and when $\overline{z_{I}^{2}}$ and $\overline{z_{I}^{2}}$ differ appreciably, which occurs when there is a considerable spread in the values of the charges of ions of one sort, localized in a given region of space, the flux Γ_{I} , in particular, may vanish.

const, the equilibrium impurity ion density has the form

$$n_{I} = \operatorname{const} n_{i}^{\bar{z}_{I}} T^{\frac{3}{2}} \mu_{iI} \left(\frac{1}{m_{i} - \bar{z}_{I}} / m_{I} \right) + \bar{z}_{I} - 1}.$$
(9)

In the most typical case $(m_{I} \ge 5m_{i})$ it follows from relation (9) that impurity ions will not collect in regions of maximum concentration if the density of the main plasma ions n_{i} increases more slowly than $T^{(\bar{z}_{I}+2)/2(\bar{z}_{I}-1)}$. For a plasma with ions of two sorts of different charge ($\bar{z} = 1$) Eq. (9) shows that light particles are concentrated in the region of maximum temperature. In particular, for a D-T-plasma, the densities of deuterium and tritium ions in equilibrium are related by the expression

$$n_T/n_D \sim T^{-3/10}$$

where it follows that the tritium ions are taken out to the periphery of the plasma filament.

We will now determine the transverse ionic thermal conductivity of a plasma with one sort of impurity ions. Assuming, as in the previous example, that $z_i = 1$, $m_I \gg m_i$ and also assuming that $\nabla_{\perp} T_i = \nabla_{\perp} T_I$, we obtain from Eq. (6)

$$\chi = \chi_i + \chi_I = \frac{2p_i}{\omega_i^2 \tau_{ii} m_i} \left[1 + \frac{n_I z_I^2}{n_i} \left(2.3 + \frac{n_I}{n_i} \sqrt{\frac{m_I}{m_i}} \right) \right].$$
(10)

The coefficient (10) in front of the square brackets is the thermal conductivity of a plasma without impurities $\chi_1^{(\circ)}$, the term proportional to $(m_I/m_I)^{1/2}$ is the correction due to the thermal conductivity of the heavy ions, and the remaining term is the correction due to the thermal conductivity of the main ions of the plasma. It is more convenient to express the transverse thermal conductivity in terms of r_{1} and r_{2}^{2}/r_{1} .

transverse thermal conductivity in terms of $z_{eff} = \sum_{\alpha} n_{\alpha} z_{\alpha}^2 / n_e$:

$$\chi = \chi_i^{(0)} \left\{ 1 + (z_{\text{eff}} - 1) \left[2.3 + (m_I/m_i)^{1/2} (z_{\text{eff}} - 1)/\overline{z_I^2} \right] \right\}.$$
(11)

Here we have used the inequality $z_{eff} \ll \overline{z_I}$, which usually holds when $\overline{z_I} \gg 1$. If we ignore the correction proportional to $(m_I/m_i)^{1/2}$, Eq. (11) can be immediately generalized to the case of a plasma with an arbitrary number of impurity ions. If, e.g., in such a plasma $z_{eff} = 4$, the transverse ionic thermal conductivity increases by a factor of approximately 8.

<u>3. Determination of the Longitudinal Friction Forces and Heat Fluxes</u>. The longitudinal heat fluxes and friction forces can be obtained from Eqs. (2), if we put $\omega_{\alpha Z} = 0$. In this case, only the last two equations will in fact remain, while the first serves merely to determine the force of friction in terms of the velocity and temperature gradient of the components. This quite complex system describes diffusion and heat transfer in a plasma with particles of arbitrary mass. However, in actual plasma, the masses of many of its components are related to one another by definite relations. In particular, we can always separate components whose particle masses are considerably less than the masses of the remaining particles. In addition, the masses of ions of one sort but with different ionization multiplicity are equal. The latter fact, as shown in Appendix 1, enables one to calculate the longitudinal components in two states: We first determine the mean values of the quantities for particles of one sort, and we then obtain the difference between the partial and average values. Using this method the general solution of Eqs. (2) for the longitudinal components of the force of friction and the heat flux can be represented in the form

$$\mathbf{R}_{\alpha z \parallel} = -\frac{n_{\alpha z} m_{\alpha} z^{2}}{\overline{z_{\alpha}^{2}}} \left\{ \sum_{\beta} \left[\frac{c_{\alpha \beta}^{(1)}}{\tau_{\alpha \beta}} \left(\mathbf{w}_{\alpha z \parallel} - \overline{\mathbf{w}}_{\beta \parallel} \right) - c_{\alpha \beta}^{(2)} \frac{\tau_{\beta}}{\tau_{\alpha \beta}} \frac{\nabla_{\parallel} T_{\beta}}{m_{\beta}} \right] + \frac{\overline{z_{\alpha}^{2}}}{z^{2}} c_{\alpha}^{(5)} \frac{\nabla_{\parallel} T_{\alpha z}}{m_{\alpha}} \right\};$$
(12)

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$$\mathbf{h}_{\alpha z \parallel} = p_{\alpha z} \tau_{\alpha} \left\{ \sum_{\beta} \left[\frac{c_{\beta \alpha}^{(2)}}{\tau_{\alpha \beta}} \left(\mathbf{w}_{\alpha z \parallel} - \overline{\mathbf{w}}_{\beta \parallel} \right) - c_{\alpha \beta}^{(3)} \frac{\tau_{\beta}}{\tau_{\alpha \beta}} \frac{\mathbf{v}_{\parallel} T_{\beta}}{m_{\beta}} \right] - \frac{\overline{z_{\alpha}^{2}}}{z^{2}} c_{\alpha}^{(6)} \frac{\mathbf{v}_{\parallel} T_{\alpha z}}{m_{\alpha}} \right\},$$
(13)

where $\tau_{\alpha}^{-1} = \sum_{\beta} \tau_{\alpha\beta}^{-1}$. The numerical coefficients $c_{\alpha\beta}^{(n)}$ in Eqs. (12) and (13) are found from the solution of the set of equations for the mean values, while $c_{\alpha}^{(n)}$ is found from the equations for the difference between the partial and mean values. In this case $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ are related by the equations

$$c_{lphaeta}^{(1)}=c_{etalpha}^{(1)},~~\sum_{eta}rac{c_{lphaeta}^{(1)}}{ au_{lphaeta}}=rac{c_{lpha}^{(4)}}{ au_{lpha}},~~\sum_{eta}rac{c_{etalpha}^{(2)}}{ au_{lphaeta}}=rac{c_{lpha}^{(5)}}{ au_{lphaeta}},~~c_{lphaeta}^{(3)}=c_{etalpha}^{(3)}.$$

The summation here, as in Eqs. (12) and (13), can only be extended to different sorts of particles.

Hence, the problem now reduces to finding the coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ for components with different particle masses. The general expressions for these quantities are quite complicated, but for light particles (at least one sort of such particles (electrons) is always present in a plasma) an explicit form of the coefficients can be obtained using an expansion with respect to the small parameter m_k/m_{α} (the index k relates to the light particles). In the zeroth approximation with respect to m_k/m_{α} the set of equations (2) splits into two independent equations: for the light component and for the heavy components with $m_{\alpha} \gg m_k$ (cf., e.g., [3, 11], where this method was used). Solution of the first of these leads to the following values of the coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ for light particles:

$$c_{k\alpha}^{(1)} = (1 + 0.24z_{k}^{*})(1 + 0.93z_{k}^{*})/\Delta_{k}, \quad c_{k\alpha}^{(2)} = 0, \quad c_{k}^{(5)} - c_{kk}^{(2)}\tau_{k}/\tau_{kk} = = 2.2z_{k}^{*}(1 + 0.52z_{k}^{*})/\Delta_{k}, \quad c_{\alpha k}^{(2)} = 1.56(1 + 1.41z_{k}^{*})(1 + 0.52z_{k}^{*})/\Delta_{k}, \quad (14)$$
$$c_{k\alpha}^{(3)} = 0, \quad c_{k}^{(6)} + c_{kk}^{(3)}\tau_{k}/\tau_{kk} = 3.9(1 + 1.41z_{k}^{*})(1 + 1.7z_{k}^{*})/\Delta_{k},$$

where $\Delta_k = (1 + 2.65z_k^*)(1 + 0.285z_k^*); z_k^* = \sum_{\alpha} n_{\alpha} \overline{z_{\alpha}^2}/n_k \overline{z_k^2}$. The summation here is carried out over all

the sorts of particles α for which $m_{\alpha} \gg m_k$. In deriving the coefficients (14) we assumed that in the majority of cases the light particles are those particles with only one possible charged state, so that $c_{kk}^{(n)}$ and $c_k^{(n)}$ occur in (12) and (13) in only certain combinations. Note that, as shown in [8-10], if the condition $T_k/m_k \gg T/m_{\alpha}$ is satisfied, the separation of the equations for the light particles is also possible when their temperature differs from the temperature of the remaining components. In this case, it is necessary to use their inherent temperature in the coefficients of the expressions for the force of friction (12) and the heat flux (13) of light particles.

The coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ for the heavy particles are determined from (2) written without the light component. The effect of the latter reduces only to taking into account the force of friction of the heavy particles on the light particles [17]. In fact, in addition to the addition of new terms in Eqs. (12) and (13), connected with the light components, this also leads to a small change in

$$c_{\alpha\beta}^{(1)} = c_{\alpha\beta}^{(1)T} + 1.77 \left(1 + 0.3 z_{k}^{*}\right) \left(m_{k}/\mu_{\alpha\beta}\right)^{1/2}/\Delta_{k},$$

where $c_{\alpha\beta}^{(1)T}$ is the coefficient calculated without taking the light component into account.

Obviously if after separating the equations for the lightest of the components in the plasma there is again a component for which $m_i \ll m_\alpha$, where $\alpha \neq i$, k (e.g., protons in a plasma with heavy impurities), the procedure for separating the equations from the system of equations (12) can be extended. In this case the coefficients in Eqs. (12) and (13) for the force of friction and the heat flux of the component will again be determined by Eqs. (14).

The coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ for the heavy components and masses that are not too different, retaining in the result of the last separations all the light components, should of course be expressed in terms of the ratio of the corresponding determinants. But this hardly makes sense since the calculation of the determinants themselves in the case of

arbitrary masses turns out to be extremely complicated even for two sorts of particles. Hence, it is best to make a numerical calculation of these quantities. Such a calculation was carried out for two heavy components. The equations for the approximate calculations of the coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ for several mass ratios corresponding to any pair of the set of impurities carbon, oxygen, iron, and tungsten, are presented in Appendix 2.

In [6] expressions for the longitudinal friction forces and heat fluxes obtained for the special case of a plasma with two sorts of ions were used to analyze the radial particle and heat transfer transverse to the magnetic surface in colloidal systems in the Pfirsh-Schlüter regime. In [11], expressions obtained for a multicomponent plasma with a large mass ratio of the ions were used for the same purpose. The expressions obtained in the present paper enable one to analyze the diffusion and heat transfer in the Pfirsh-Schlüter regime for a multicomponent plasma with impurities of arbitrary mass in arbitrary charged states.

<u>Appendix 1.</u> Summation of Eqs. (2) over the charged states when determining the longitudinal transfer properties is possible due to the following property of the coefficients of the equations:

$$G^{(n)}_{lpha z eta \zeta} = I_{lpha z} I_{eta \zeta} \overline{G}^{(n)}_{lpha eta}, ext{ where } I_{lpha z} = n_{lpha z} z^2 / n_{lpha} \overline{z^2_{lpha}}, \quad \overline{G}^{(n)}_{lpha eta} = \sum_{z, \zeta} G^{(n)}_{lpha z eta \zeta},$$

Using these relations, Eqs. (2b) and (2c) in the case of interest ($\omega_{\alpha Z}$ = 0) can be written, after summation over ζ , in the form

$$\frac{5}{2} n_{\alpha z} \nabla_{\parallel} T_{\alpha z} = I_{\alpha z} \sum_{\beta} \left\{ \frac{5}{2} \frac{\mu_{\alpha\beta}}{m_{\alpha}} \,\overline{G}_{\alpha\beta}^{(2)} \left(\mathbf{w}_{\alpha z} - \overline{\mathbf{w}}_{\beta} \right) + \overline{G}_{\alpha\beta}^{(4)} \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}} - \overline{G}_{\alpha\beta}^{(5)} \frac{\overline{\mathbf{h}}_{\beta}}{p_{\beta}} + \frac{\mu_{\alpha\beta}}{T} \left(\overline{G}_{\alpha\beta}^{(6)} \frac{\mathbf{r}_{\alpha z}}{p_{\alpha z}} - \overline{G}_{\alpha\beta}^{(7)} \frac{\overline{\mathbf{r}}_{\beta}}{p_{\beta}} \right) \right\}, \tag{A1}$$
$$0 = I_{\alpha z} \sum_{\beta} \left\{ \frac{35}{2} \left(\frac{\mu_{\alpha\beta}}{m_{\alpha}} \right)^2 \overline{G}_{\alpha\beta}^{(3)} \left(\mathbf{w}_{\alpha z} - \overline{\mathbf{w}}_{\beta} \right) + 7 \frac{\mu_{\alpha\beta}}{m_{\alpha}} \left(\overline{G}_{\alpha\beta}^{(6)} \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}} - \overline{G}_{\alpha\beta}^{(7)} \frac{\overline{\mathbf{h}}_{\beta}}{p_{\beta}} \right) + \overline{G}_{\alpha\beta}^{(8)} \frac{m_{\alpha} \mathbf{r}_{\alpha z}}{T p_{\alpha z}} - \overline{G}_{\alpha\beta}^{(9)} \frac{m_{\beta} \overline{\mathbf{r}}_{\beta}}{T p_{\beta}} \right\},$$

where $\mathbf{w}_{\alpha} = \sum_{z} I_{\alpha z} \mathbf{w}_{\alpha z};$ $\mathbf{h}_{\alpha} = p_{\alpha} \sum_{z} I_{\alpha z} \mathbf{h}_{\alpha z} / p_{\alpha z};$ $\mathbf{r}_{\alpha} = p_{\alpha} \sum_{z} I_{\alpha z} \mathbf{r}_{\alpha z} / p_{\alpha z}$ are the mean values of the quantities $\mathbf{w}_{\alpha z}$, $\mathbf{h}_{\alpha z}$, $\mathbf{r}_{\alpha z}$ for particles of the sort α .

Summing Eqs. (Al) over z, we arrive at the following set of equations:

$$\frac{5}{2} n_{\alpha} \nabla_{\parallel} T_{\alpha} = \sum_{\beta} \left\{ \frac{5}{2} \frac{\mu_{\alpha\beta}}{m_{\alpha}} \overline{G}_{\alpha\beta}^{(2)} \left(\overline{\mathbf{w}}_{\alpha} - \overline{\mathbf{w}}_{\beta} \right) + \overline{G}_{\alpha\beta}^{(4)} \frac{\overline{\mathbf{h}}_{\alpha}}{p_{\alpha}} - \overline{G}_{\alpha\beta}^{(5)} \frac{\overline{\mathbf{h}}_{\beta}}{p_{\beta}} + \frac{\mu_{\alpha\beta}}{T} \left(\overline{G}_{\alpha\beta}^{(6)} \frac{\overline{\mathbf{r}}_{\alpha}}{p_{\alpha}} - \overline{G}_{\alpha\beta}^{(7)} \frac{\overline{\mathbf{r}}_{\beta}}{p_{\beta}} \right) \right\},$$

$$0 = \sum_{\beta} \left\{ \frac{35}{2} \left(\frac{\mu_{\alpha\beta}}{m_{\alpha}} \right)^{2} \overline{G}_{\alpha\beta}^{(3)} \left(\overline{\mathbf{w}}_{\alpha} - \overline{\mathbf{w}}_{\beta} \right) + 7 \frac{\mu_{\alpha\beta}}{m_{\alpha}} \left(\overline{G}_{\alpha\beta}^{(6)} \frac{\overline{\mathbf{h}}_{\alpha}}{p_{\alpha}} - \overline{G}_{\alpha\beta}^{(7)} \frac{\overline{\mathbf{h}}_{\beta}}{p_{\beta}} \right) + \overline{G}_{\alpha\beta}^{(8)} \frac{m_{\alpha}\overline{\mathbf{r}}_{\alpha}}{Tp_{\alpha}} - \overline{G}_{\alpha\beta}^{(9)} \frac{m_{\beta}\overline{\mathbf{r}}_{\beta}}{Tp_{\beta}} \right\}.$$
(A2)

Equations (A2) are similar to the last two equations of system (2), but the number of equations is considerably less here. Formal solution of this equation enables one to express $\bar{\mathbf{r}}_{\alpha}$ and $\bar{\mathbf{h}}_{\alpha}$ in terms of $\bar{\mathbf{w}}_{\alpha}$ and $\nabla_{||}T_{\alpha}$, and after substituting these quantities into the right side of Eq. (2a) one can determine the values of the constant $c_{\alpha\beta}^{(n)}$. The calculation of the mean values of $\bar{\mathbf{r}}_{\alpha}$ and $\bar{\mathbf{h}}_{\alpha}$ is not, however, a complete solution of the problem, since to describe the behavior of the plasma it is necessary to know the diffusion and heat transfer for each of the charged states of the ion. In order to calculate these quantities we will divide Eqs. (A1) into $I_{\alpha z}$ and subtract them term by term from the corresponding equations of system (A2). As a result we have

$$\frac{5}{2}n_{\alpha}\left(\mathbf{\nabla}_{\parallel}T_{\alpha} - \frac{\overline{\mathbf{z}_{\alpha}^{2}}}{z^{2}}\mathbf{\nabla}_{\parallel}T_{\alpha z}\right) = \left(\overline{\mathbf{w}}_{\alpha} - \mathbf{w}_{\alpha z}\right)S_{\alpha}^{(2)} + \left(\frac{\overline{\mathbf{h}}_{\alpha}}{p_{\alpha}} - \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}}\right)S_{\alpha}^{(4)} + \left(\frac{\overline{\mathbf{r}}_{\alpha}}{p_{\alpha}} - \frac{\mathbf{r}_{\alpha z}}{p_{\alpha z}}\right)\frac{m_{\alpha}}{T}S_{\alpha}^{(6)},$$

$$0 = \left(\overline{\mathbf{w}}_{\alpha} - \mathbf{w}_{\alpha z}\right)S_{\alpha}^{(3)} + 7\left(\frac{\overline{\mathbf{h}}_{\alpha}}{p_{\alpha}} - \frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}}\right)S_{\alpha}^{(6)} + \left(\frac{\overline{\mathbf{r}}_{\alpha}}{p_{\alpha}} - \frac{\mathbf{r}_{\alpha z}}{p_{\alpha z}}\right)\frac{m_{\alpha}}{T}S_{\alpha}^{(8)},$$
(A3)

where $S_{\alpha}^{(2)} = \frac{5}{2} \sum_{\beta} \frac{\mu_{\alpha\beta}}{m_{\alpha}} \overline{G}_{\alpha\beta}^{(2)}; \quad S_{\alpha}^{(3)} = \frac{35}{2} \sum_{\beta} \left(\frac{\mu_{\alpha\beta}}{m_{\alpha}} \right)^2 \overline{G}_{\alpha\beta}^{(3)}; \quad S_{\alpha}^{(4)} = \sum_{\beta} \overline{G}_{\alpha\beta}^{(4)}; \quad S_{\alpha}^{(6)} = \sum_{\beta} \frac{\mu_{\alpha\beta}}{m_{\alpha}} \overline{G}_{\alpha\beta}^{(6)}; \quad S_{\alpha}^{(8)} = \sum_{\beta} \overline{G}_{\alpha\beta}^{(8)}.$ The

TABLE 1

	C-0	Fe-W	O-Fe	C-Fe	o-w	C-W
m_i/m_I	0,750	0,304	0,286	0,214	0,087	0,065
$\begin{array}{c} P_1 \\ c_{ii}^{(1)} P_2 \\ P_3 \end{array}$	0,74 0,09 0,86	0,63 0,11 0,53	0,63 0,11 0,53	0,63 0,10 0,49	0,62 0,09 0,45	0,62 0,09 0,45
$\begin{array}{c} & & P_1 \\ c_{iI}^{(1)} & P_2 \\ & & P_3 \end{array}$	0,77 0,10 0,87	0,55 0,27 0,63	0,54 0,27 0,62	0,48 0,29 0,57	0,38 0,30 0,50	0,36 0,31 0,49
$\begin{array}{c} & P_1 \\ & P_2 \\ c_{II}^{(1)} & P_2 \\ & P_3 \end{array}$	0,84 0,12 0,88	0,84 0,36 0,65	0,84 0,36 0,63	0,84 0,38 0,58	0,84 0,33 0,49	0,84 0,31 0,48
$\overbrace{\begin{array}{c} & P_1 \\ c_{ii}^{(2)} & P_2 \\ & P_3 \end{array}}^{P_1}$	0,51 0,07 0,92	0,38 0,11 0,55	0,38 0,10 0,51	0,40 0,06 0,32	0,47 0,01 0,10	0,49 0,01 0,10
$\begin{array}{c} & P_{1} \\ c_{iI}^{(2)} & P_{2} \\ & P_{3} \end{array}$	0,37 0,07 1,00	0,07 0,27 1,16	0,08 0,27 1,15	-0,12 0,27 1,15	-0,11 0,18 1,12	-0,09 0,14 1,10
$\begin{matrix} & P_{1} \\ c_{1i}^{(2)} & P_{2} \\ & P_{3} \end{matrix}$	0,75 0,04 0,80	1,14 0,02 0,10	1,16 0,02 0,10	1,23 0,01 0,08	1,29 0,00 0,00	1,29 0,00 0,00
$\begin{array}{c} & P_{1} \\ c_{II}^{(2)} & P_{2} \\ & P_{3} \end{array}$	$\begin{array}{c c} 0,59 \\ 0,04 \\ 0,86 \end{array}$	0,58 0,12 13,00	$ \begin{array}{c} 0,59 \\ -0,45 \\ 8,00 \end{array} $	$ \begin{array}{r} 0,59 \\ -0,30 \\ 2,90 \end{array} $	$0,58 \\ -0,32 \\ 1,40$	$0,58 \\ -0,31 \\ 1,20$
$\begin{array}{c} & P_1 \\ c_i^{(5)} & P_2 \\ P_3 \end{array}$	$ \begin{array}{c c} 0,75 \\ -0,17 \\ 1,09 \end{array} $	1,17 0,78 1,36	1,19 0,82 1,37	$ \begin{array}{ c c c c } 1,26 \\ -0,95 \\ 1,43 \\ \end{array} $	1,37 1,19 1,56	1,39 -1,26 1,59
$\begin{matrix} & P_1 \\ c_I^{(5)} & P_2 \\ & P_3 \end{matrix}$	0,59 -0,15 1,09	$ \begin{array}{c} 0,58 \\ -0,52 \\ 1,26 \end{array} $	$\begin{array}{c} 0,58 \\0,53 \\ 1,25 \end{array}$	$ \begin{array}{c c} 0,58 \\ -0,56 \\ 1,21 \end{array} $	$\begin{array}{c} 0,57 \\ -0,50 \\ 0,95 \end{array}$	$0,57 \\ -0,46 \\ 0,86$
$\begin{matrix} P_1 \\ c_{1i}^{(3)} & P_2 \\ P_3 \end{matrix}$	1,54 0,31 1,28	2,57 4,00 3,10	2,69 4,65 3,29	3,41 8,80 4,10	6,79 3 8,00 6,80	8,00 -52,00 7,64
$\begin{array}{c} & P_{1} \\ c_{iI}^{(3)} & P_{2} \\ & P_{3} \end{array}$	$ \begin{array}{r} 1,35 \\ -0,28 \\ 1,26 \end{array} $	0,64 0,17 1,43	0,56 0,07 0,90	0,20 1,20 6,00	0,50 4,00 6,00	0,55 4,50 6,60
$\begin{array}{c} & P_1 \\ c_{II}^{(3)} & P_2 \\ & P_3 \end{array}$	$ \begin{array}{c c} 1,29 \\ -0,30 \\ 1,25 \end{array} $	1,29 1,67 2,02	1,29 -1,74 2,05	$ \begin{array}{c c} 1,29 \\ -2,02 \\ 2,16 \end{array} $	$\begin{array}{c c} 1,27 \\ -2,10 \\ 2,00 \end{array}$	1,28 -1,96 1,84
$ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} $	3,17 0,64 1,16	5,32 5,05 1,87	5,49 5,53 1,93	$ \begin{array}{r} 6,30 \\ -8,11 \\ 2,20 \end{array} $	8,85 19,10 3,10	9,55 23,00 3,31
$\begin{array}{c} & P_{1} \\ c_{I}^{(6)} & P_{2} \\ P_{3} \end{array}$	$ \begin{array}{c c} 2,61 \\ -0,53 \\ 1,15 \end{array} $	$ \begin{array}{c c} 2,61 \\ -2,13 \\ 1,63 \end{array} $	$2,61 \\ -2,21 \\ 1,65$	2,61 2,47 1,70	$ \begin{array}{c c} 2,60 \\ -2,51 \\ 1,53 \end{array} $	2,60 -2,36 1,41

solution of Eqs. (A3) for $\overline{h}_{\alpha}/p_{\alpha}-h_{\alpha Z}/p_{\alpha Z}$ can be written in the form

$$\frac{\mathbf{h}_{\alpha z}}{p_{\alpha z}} = \frac{\overline{\mathbf{h}}_{\alpha}}{p_{\alpha}} + c_{\alpha}^{(6)} \left(\mathbf{w}_{\alpha z} - \overline{\mathbf{w}}_{\alpha} \right) + \frac{\tau_{\alpha}}{\tau_{\alpha \alpha}} c_{\alpha}^{(5)} n_{\alpha} \left(\frac{\overline{z_{\alpha}^{2}}}{z^{2}} \nabla_{\parallel} T_{\alpha z} - \nabla_{\parallel} T_{\alpha} \right), \tag{A4}$$

where

$$c_{\alpha}^{(5)} = \frac{5}{2} \frac{S_{\alpha}^{(8)}}{D_{\alpha}} \frac{\tau_{\alpha\alpha}}{\tau_{\alpha}}; \ c_{\alpha}^{(6)} = \frac{S_{\alpha}^{(2)} S_{\alpha}^{(8)} - S_{\alpha}^{(3)} S_{\alpha}^{(6)}}{D_{\alpha}}; \ D_{\alpha} = S_{\alpha}^{(4)} S_{\alpha}^{(8)} - 7 \left(S_{\alpha}^{(6)}\right)^{2}.$$

The solution of (A3) in the form (A4) together with the solution of the system of equations (A2) for \overline{h}_{α} leads as a result to Eq. (13). Substituting the solutions of these equations into Eq. (2a) we can represent $R_{\alpha z}$ in the form (12), where the coefficient $c_{\alpha}^{(4)}$ is given by the expression

$$c_{\alpha}^{(4)} = 1 - \frac{2}{5} \left[\left(S_{\alpha}^{(2)} \right)^2 S_{\alpha}^{(8)} - 2 S_{\alpha}^{(2)} S_{\alpha}^{(3)} S_{\alpha}^{(6)} + \left(S_{\alpha}^{(3)} \right)^2 S_{\alpha}^{(6)} / 7 \right] \tau_{\alpha} / D_{\alpha} \tau_{\alpha \alpha}.$$

<u>Appendix 2.</u> The coefficients $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ in the case of two heavy components are most conveniently calculated from the approximate equation

$$c = P_1 + P_2/(Z_{iI} + P_3),$$

where $Z_{iI} = n_I z_I^2 / n_i z_i^2$. The index I corresponds to the heaviest component of the plasma. The values of the constants P_n for determining $c_{\alpha\beta}^{(n)}$ and $c_{\alpha}^{(n)}$ in a plasma consisting of two sorts of impurity ions (any pair from the set carbon, oxygen, iron, and tungsten) are given in Table 1. The error that arises from the use of the approximate formula in the majority of cases does not exceed 2-3%.

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